Fast switching current detection at low critical currents

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A pulse-and-hold technique is used to measure the switching of small critical current Josephson junctions. This technique allows one to achieve a good binary detection and therefore measure switching probabilities. The technique overcomes limitations on simple square pulses and allows for the measurement of junctions with critical currents of the order of 10nA with bias pulses of the order of 100ns. A correlation analysis of the switching events is performed to show how the switching probability depends on the wait time between repeated bias pulses.

I. INTRODUCTION

In the last several years there has been increasing interest in the quantum behavior of small capacitance Josephson junction circuits. Progress in this field has led to quantum control experiments, which use a state preparation, evolution, and read-out, to demonstrate that the electrodynamics of these circuits behaves quantum mechanically.[1, 2, 3, 4, 5] Engineering circuits so as to isolate two energy levels and studying their quantum time evolution is interesting in the context of the long-term goal of building a solid-state, scalable quantum bit processor. In many of these quantum control experiments, the measurement process, or the projection to a basis state and readout of that state, is achieved by quickly measuring the critical current of a Josephson junction. In this paper, we examine this detection technique in some detail and demonstrate how it might be extended to shorter measurement times and lower critical currents.

The quantum nature of the Josephson junction was first probed by studies of the fluctuations in the switching current, or current at which the junction jumps to the finite voltage state [6, 7]. The technique used to measure the switching current, originally introduced by Fulton and Dunkleberger [8], was based on a rapid ramping of the bias current. With each ramping event, the junction was switched to the finite voltage state. By fast measurement of the voltage across the junction, one could infer at what current the junction switched, and build a switching histogram, or distribution of the switching probability vs. bias current. The Quantronics group introduced a new technique [9, 10] with short, square pulses ($\simeq 500$ ns duration) of the current to a value near the critical current ($\simeq 0.5\mu$ A) of the junction. The pulse would cause the junction to switch with some probability. The switching of the junction is measured in a binary way (i.e. yes or no) with each pulse. By repeating the pulse many times in a sequence, one can determine the switching probability for that particular amplitude and duration of the pulse. This pulse method was successfully used as the readout in a quantum control experiment.[2]

Both the ramping method and methods based on simple square pulses, are limited in measurement speed and accuracy by the rate of increase of the measured voltage across the junction, dV_J/dt . This rate may be limited by the band width of the amplifier, or by low pass filters used to protect the junction from high frequency noise. However, in the absence of such limitations, the junction voltage will rise at a rate $dV_J/dt = V_{bias}/R_{bias}C_{Leads}$. The speed of the measurement will then be determined by the time needed for the junction voltage to rise above the noise level, so that a switching event can be triggered. If, however, we use a more complex shape of the voltage pulse, we can circumvent this limitation by exploiting the non-linear character of the Josephson tunnel junction I-V curve and the latching nature of the circuit. We have developed such a method which is described below. This method has also recently been reported by the Delft group [4], and the Quantronics group [11]. Pulse methods have also been used for studying switching properties of magnetic junctions [12].

II. EXPERIMENTAL METHOD

We have used a pulse and hold method to study the switching current of a DC SQUID. The SQUID was chosen because the level of the critical current could be adjusted by applying a magnetic flux to the SQUID loop, allowing us to explore how the method works for a wide range of critical currents, from 270-30nA with one sample. The capacitance of the parallel combination of the two Josephson junctions is roughly $C \simeq 7.2$ fF, and the normal state resistance of the SQUID (two junctions in parallel) is $R \simeq 1.15k\Omega$, giving a bare critical current value $I_{c0} = 273$ nA. When measuring the DC IV curve of the junction at zero magnetic field, we find switching at roughly 80% of this value.

The schematic of the measurement circuit and a picture of the sample are shown in Figure 1. The SQUID sample was biased from an arbitrary waveform generator (AWG) through an attenuator, a bias resistor and a low-pass filter. The connection from room temperature to the sample in a dilution refrigerator with a base temperature of about 25 mK, was realized via twisted pair cables of constantan wire of length 2.1 m and resistance 135Ω per wire. No special low-pass filters were implemented at low temperatures.

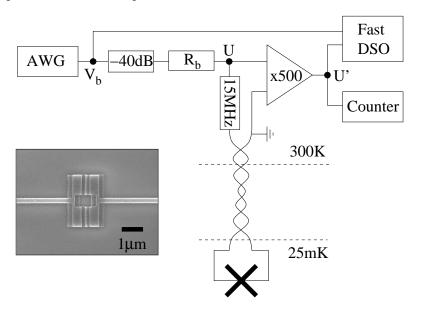


FIG. 1: The experimental setup and a micro graph of the sample. AWG stands for arbitrary waveform generator and DSO stands for digital sampling oscilloscope. The picture shows a micrograph of the sample.

The shape of the probe pulse as shown in fig. 2a has three parts. A very short "switch pulse" with a duration τ_p is applied, which may cause the junction to switch from the supercurrent branch to the finite voltage state. This switch pulse can be much shorter than the time needed to charge up the capacitance at the input of the voltage amplifier, and no voltage rise is registered from this pulse alone. The switch pulse is immediately followed by a hold level for a time τ_h , so that if the junction has switched to the finite voltage state, it will not "retrap" to the zero voltage branch of the I-V curve. The duration of the hold level is set just long enough for the junction voltage to rise above the noise level and trigger a counter to record a switching event. When calibrating the hold level, one can check that a pulse consisting of the hold level itself, causes no switching events. Following the hold level, a wait time τ_w is programmed in the AWG. τ_w must be long enough for the junction to relax to the equilibrium state. The requirements on τ_w are determined by a correlation analysis of the switching sequence of many pulses as described below.

Figure 2b shows 8 typical oscilloscope traces overlaid on top of one-another, of the junction voltage U measured at the top of the cryostat. By adjusting the hold level and duration, a clear separation between the switch and no-switch event can be obtained. All the signals that rise above a trigger level of $40\mu V$ are recorded as a switching event by the counter. Typically a burst of 10^4 identical pulses is applied to the sample and the number of switching events N is counted. The switching probability is then $P = N/10^4$. It is important that a clear separation exists in the voltage response from switching and non-switching events, as shown in fig. 2b. This separation is a necessary condition for a binary detector of switching and the use of event counting statistics for determining the switching probability. We find that with our measurement set-up, simple square pulses will not give this clear event separation, and ambiguities arise between switching events due to noise and those due to actual junction switching in the final moments of the square pulse duration.

In a typical measurement, after the hold voltage level and τ_h are adjusted, the height of the switch pulse is initially chosen so that P=0 after 10^4 pulses. The amplitude of the switch pulse is then slightly increased while the other parameters of the pulse are kept constant and the burst sequence is repeated, recording a new value of P. This procedure is repeated until the switching probability reaches P=1. In this way a P(I) curve can be measured, as seen in fig. 2c. Taking the derivative of this curve we arrive at the switching current distribution shown in fig. 2d. Here, the current level is simply given by $I=V_{switch}/R_b$, where V_{switch} is the voltage amplitude of the switch pulse as programmed in to the AWG, divided by the attenuation factor. This method of determining the current is valid for $\tau_p > 1\mu s$. In the experimental set-up shown in fig. 1 the shortest switch pulse attainable that was reasonably square was of order $\tau_p = 100 \text{ns}$, as measured by an oscilloscope at the bottom of the cryostat when the system was at room temperature. Shorter pulses suffered from too much dispersion in the twisted pairs, which limited the rise and fall

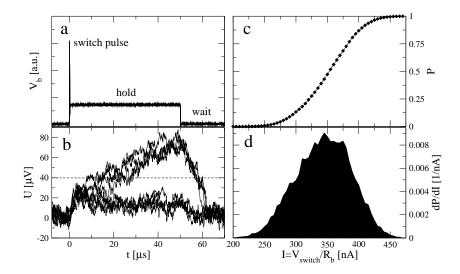


FIG. 2: (a) The probe pulse consisting of the switch pulse (duration τ_p), hold level (duration τ_h), and wait time (duration τ_w) between consecutive pulses. (b) Response of the sample for 8 identically applied pulses. (c) By successively increasing the switch pulse the switching probability P(I) is obtained. The derivative of the P(I) curve leads to the switching current distribution (d).

times of the pulses to 20ns. Dispersion also caused a second, slow rise of order 1μ s, which made the actual amplitude of the pulse at the junction dependent on the pulse duration. For this reason, it is difficult to calibrate the current level I of the shortest pulses. However, our analysis of the data relies only on a figure of merit for the detector, which is the ratio of the width of the switching distribution to the pulse height at the maximum of the distribution, $\Delta I/I$. Thus, this unknown calibration factor for short pulses due to dispersion in the twisted pairs, falls away in the final analysis.

The value of the bias resistor, R_b shown in fig. 1 was chosen between $1\mathrm{k}\Omega$ and $20\mathrm{k}\Omega$, depending on the level of the switching current in the range 220nA to 10nA respectively. It is desirable to make R_b as small as possible for a fast rise of the measured junction voltage. However R_b must be large enough so that the junction voltage will not retrap immediately after switching, during the hold time. We find that it is necessary to increase R_b for smaller switching currents, and that the detailed nature of the junction I-V curve can play an important role in the choice of R_b . For example, resonances with high frequency electromagnetic modes of the environment can give parasitic peaks in the junction I-V curve, which inhibit the latching property of the circuit when R_b is too small. Such resonant Cooper pair tunneling at finite voltage with associated excitation of the electromagnetic environment, is most likely the major source of dissipation in this detector, where quasiparticle tunneling rates are quite suppressed at these temperatures and voltages, well below the gap energy. Note that we pull the voltage back to zero before the capacitance of the bias leads is fully charged to the quiescent operating point of the circuit. However, the value of R_b is chosen such that the junction never reaches the gap voltage $V_{2\Delta} \approx 400 \mu V$ and is typically kept below $80 \mu V$, as can be seen in fig. 2b.

In order to study how the switching or not-switching of the junction effects subsequent measurements in the pulse sequence, we need to not only count switching events, but also to store the actual binary sequence of switching in temporal order. A fast digital sampling oscilloscope (DSO) with deep memory was used to capture and store the entire burst sequence of 10^4 pulses from the AWG, and the amplified sample voltage U'. The data could be analyzed to reconstruct the binary sequence of switching events. Correlation functions were studied as a function of the wait time between pulses, τ_w .

III. RESULTS

The measurement scheme described above can be used to detect the energy eigenstate of a quantum circuit. For this purpose it is desirable to have a resolution $\Delta I/I$ which is as narrow as possible, and a measurement time τ_p as short as possible. Figure 3 shows the dependence of $\Delta I/I$ for different values of τ_p . The magnetic field of the SQUID was tuned such that the critical current for this set of data was calculated to be $I_{c0} \approx 28nA$. The switching current however was $\simeq 11$ nA. As one can see in fig. 3, even for the short pulses with $\tau_p = 100ns$, the resolving power of the detector is better than 1nA. The increase of $\Delta I/I$ for shorter pulses was observed over the entire range of critical currents measured with this SQUID. We also found essentially the same qualitative behavior for a sample with an

identical SQUID shun bias resistor bonded r

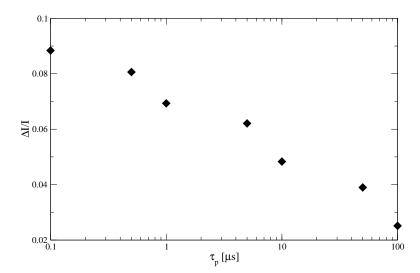


FIG. 3: Dependence of the resolving power $\Delta I/I$ of the detector as a function of the pulse width τ_p for a sample with $I_{switch} \simeq 11 \mathrm{nA}$.

If the detector is working in an ideal way, each switching event should be statistically independent. We can check for this independence by making a correlation analysis of the binary sequence of switching events, ordered as they were measured. We captured the entire sequence and analyzed the data, assigning a value $Y_i = 0$ for no switch, and $Y_i = 1$ for switch, in chronological order with the index i. We study the auto correlation function of this sequence which is defined as

$$r_k = \frac{\sum_{i=1}^{N_0 - k} (Y_i - \bar{Y})(Y_{i+k} - \bar{Y})}{\sum_{i=1}^{N_0} (Y_i - \bar{Y})^2}.$$
 (1)

Here, $N_0 = 10^4$ is the total number of pulses and Y = P(I) is the switching probability for the particular pulse sequence applied. The auto correlation, r_1 , with lag k = 1, is shown in fig. 4 as a function of the wait time between measurement pulses, τ_w . For each data point in fig. 4, the pulse was adjusted such that $\bar{Y} = P(I) = 0.5 \pm 0.05$, and the data shown is for the case zero magnetic field, or a large critical current, $I_{c0} = 270$ nA. Here we clearly see an exponential decay of the correlation between switching events which are nearest-neighbors in time. The characteristic time for decay of this correlation is 17.2μ s. Thus, we must program a wait time $\tau_w \gg 17.2\mu$ s to avoid measurement errors in our detector.

One should note that in principle, negative correlations, or anti-correlations are possible as defined by equation 1. Negative correlation would correspond to the a probe pulse that does not switch the junction, increasing the probability that the probe pulse will result in a switching event. Negative correlations might result if, for example, the dissipation during measurement were larger for non-switching than for switching events. We have performed circuit simulations of the switching process with P-spice[13] by modeling the junction as a voltage controlled switch, a standard P-spice component. Depending on the parameters of the simulation, we find that especially for simple square pulses, it is possible to realize the case where a non-switching event causes more power dissipation in the circuit, because a high current level is sustained during the entire pulse if the junction does not switch. However, we expect positive correlation for the pulse and hold technique, because in this technique we strive to make the high-current switch pulse as short as possible so that the detector can decohere the system and the quantum measurement can be as fast as possible. The power dissipation will predominantly occur during the hold time, which only is necessary to "get the answer out".

As can be seen, there is a significant correlation between adjacent pulses for wait pulses shorter than $40\mu s$. The inset of Figure 4 shows the correlation as a function of the lag time $\Delta t = (\tau_p + \tau_h + \tau_w)k$. We note that already at lag k=2 the correlation function is within the noise level $r_k \in -0.02, 0.02$. The random nature of r_k vs. k indicates that there is no periodic signal effecting the switching process. For smaller switching currents, however, we have noticed the presence of 50Hz pick-up on the measurements, which clearly shows up in the correlation function plotted vs. the lag time, Δt , as a periodic signal with period 20ms.

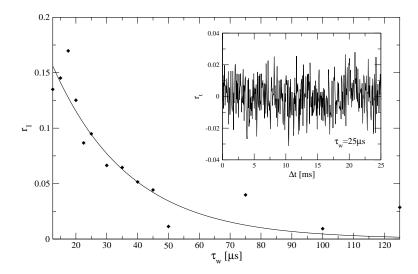


FIG. 4: Auto correlation for switching events for a sample with $I_{c0} = 270$ nA where $\tau_p = 1\mu$ s and $\tau_h = 20\mu$ s. The solid line is a fit to the data which gave $r_1 = 0.26exp(-0.04\tau_w)$. The inset shows the correlation as a function of the lag between pulses.

IV. DISCUSSION

In the DC I-V curves of the junction (not shown here) for lower critical current, we clearly see a finite slope of the supercurrent branch, due to phase diffusion. At high frequency (f > 5 MHz), the impedance of our lossy twisted pairs is essentially 50 Ω . For this shunt impedance, we expect a junction Q factor $Q = \sqrt{2\pi R^2 C I_{c0}/\Phi_0} = 0.12 << 1$, which becomes smaller as the critical current of the junction is suppressed. In this overdamped limit phase diffusion is expected at finite temperature, or in the presence of external noise [14]. At the lowest critical current, we find that the switching distributions and the DC I-V curves are essentially unchanged for temperatures less than 400mK. Such behavior is consistent with the fact that we have no low temperature filters protecting the junction. Thus, we expect that the dynamics of our junction with such low critical current, has to be described by overdamped classical dynamics of the Josephson phase in the presence of excess thermal noise. In this case the switching of the circuit is a complicated process of switching between two dynamical states of the Josephson phase, over a "dissipation barrier" resulting from a frequency dependent damping of the junction [15]. The switching occurs when a critical velocity of the phase particle is reached, and the switching process is between two dynamical states, a slow phase diffusion state, and a faster damped free-running state. It is interesting to note that reasonably good resolving power of switching current detection can be achieved in this overdamped, diffusive regime and an interesting questions arises as to weather or not a quantum detector can be designed in this regime of switching. On the one hand fast measurement might be possible, because we can always increase the switch pulse amplitude in order to more quickly drive the system to it's critical velocity. On the other hand, the actual time for the measurement is not so clearly defined in this regime, and excessive back action from long measurement time would be disadvantageous. To our knowledge, no theoretical attempts have been made to describe quantum measurement from a detector which operates in this regime. Numerical simulations are in progress to help clarify these issues.

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^[1] Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Nature 398, 786 (1999).

^[2] D. Vion et al., Science **296**, 886 (2002).

J. M. Martinis, S. Nam, and J. A. and C. Urbina, Phys. Rev. Lett 89, 117901 (2002).

^[4] I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, Science 299, 1869 (2003).

- [5] T. Duty et al., Phys. Rev. B 69, 140503 (2003).
- [6] R. F. Voss and R. A. Webb, *Phys. Rev. Lett.* 47, 265 (1981).
- [7] M. H. Devoret et al., in Quantum Tunnelling in Condensed Media, edited by Y. Kagan and A. J. Leggett (Elsevier Science, Amsterdam, 1992), Chap. 6, pp. 313–345.
- [8] T. A. Fulton and L. N. Dunkleberger, Phys. Rev. B 9, 4760 (1973).
- [9] A. Cottet et al., Physica C 367, 197 (2002).
- [10] A. Cottet et al., in International Workshop on Superconducting Nano-electronic Devices, edited by J. Pekola, B. Ruggiero, and P. Silvestrini (Kluwer Academic, Plenum Publishers, New York, 2002), pp. 73–86.
- [11] D. Vion, Private communication.
- [12] R. H. Koch et al., Phys. Rev. Lett. 81, 4512 (1998).
- [13] Available free on the internet at www.orcad.com.
- [14] J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. B 35, 4682 (1987).
- [15] P. Joyez et al., Journal of Superconductivity 12, 757 (1999).